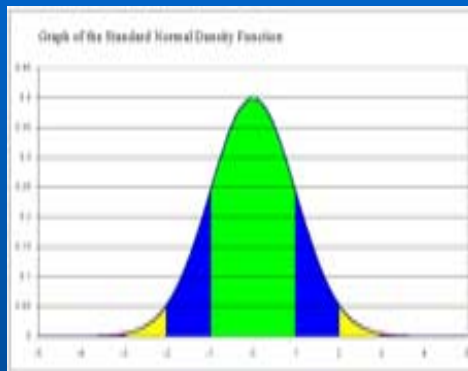
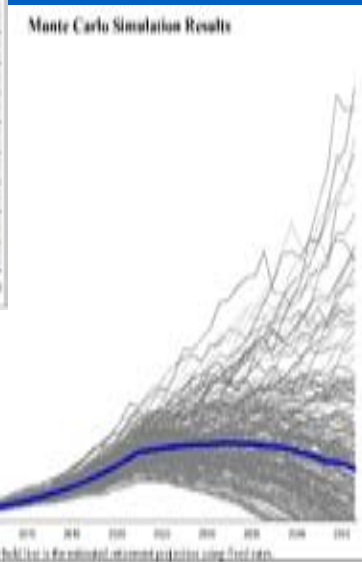


# Introduction to Monte Carlo

## Probability Based Modeling Concepts



$$g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



Mark Snodgrass  
Money Tree Software

# What is Monte Carlo?

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Monte Carlo Simulation is the currently accepted term for a technique used by mathematicians and engineers to find probable answers to highly complex and unpredictable equations. A large number of random trials are run. Patterns in the trials outcomes show the most likely range and concentration of results.

# How does Monte Carlo work?

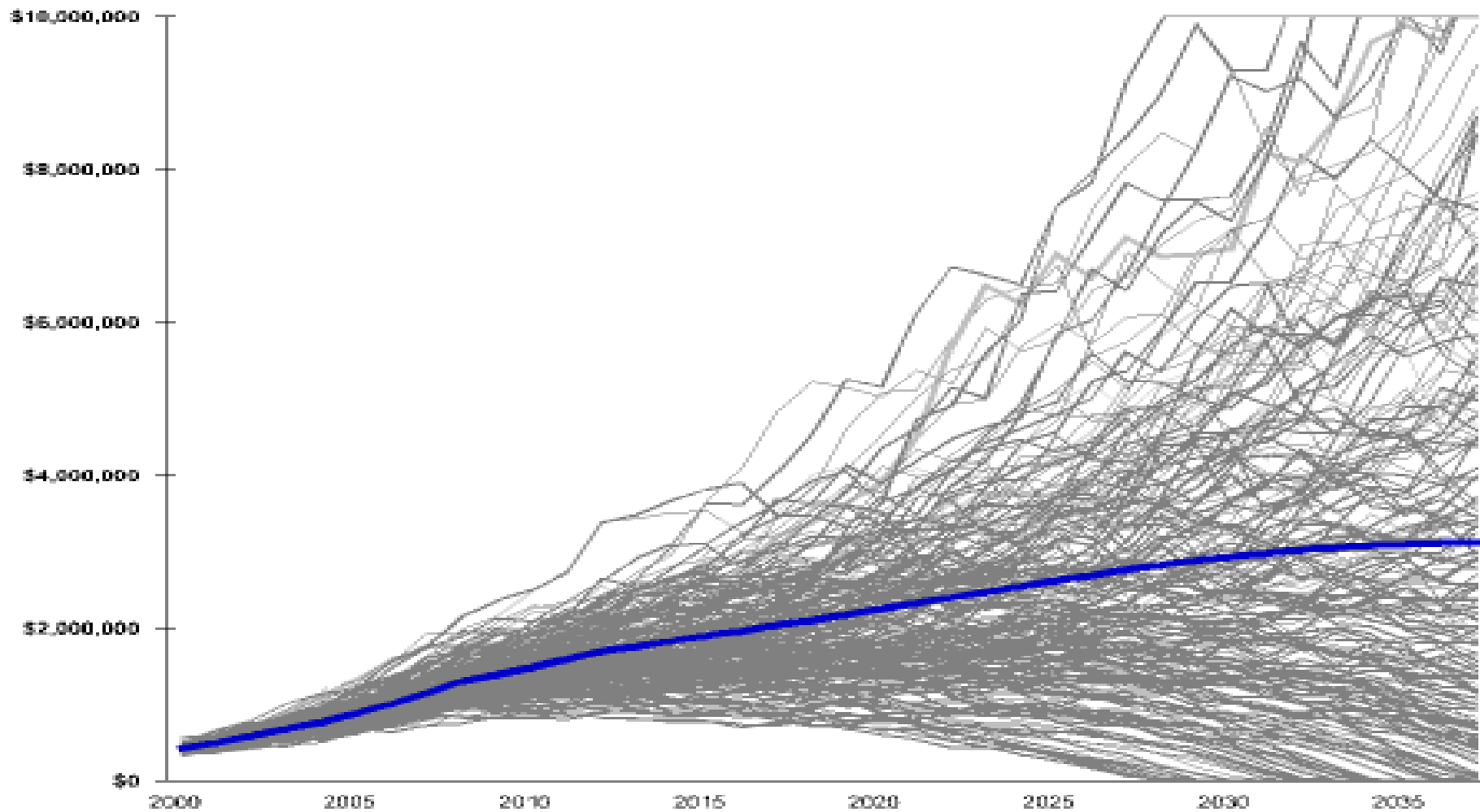
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- **Mathematical Models are Used to Reflect Future Reality**
- **Variables in the Model allow for Future Uncertainty**
- **Probability Concepts Create Random Trials in the Model**
- **Large Number of Random Trials are Run for Analysis**
- **Patterns of Results Demonstrate Trends and Certainty**
- **Statistical Results Measure Distribution and Range**
- **Graphs Help Illustrate Variability and Show Patterns**

# 1000 Simulation Results

Results of 1000 Simulations: Percentage of projections above zero 84%  
Retirement Projection Estimate \$3,123,022  
Minimum Monte Carlo projection \$0  
Average Monte Carlo projection \$3,165,938  
Maximum Monte Carlo projection \$20,351,776

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

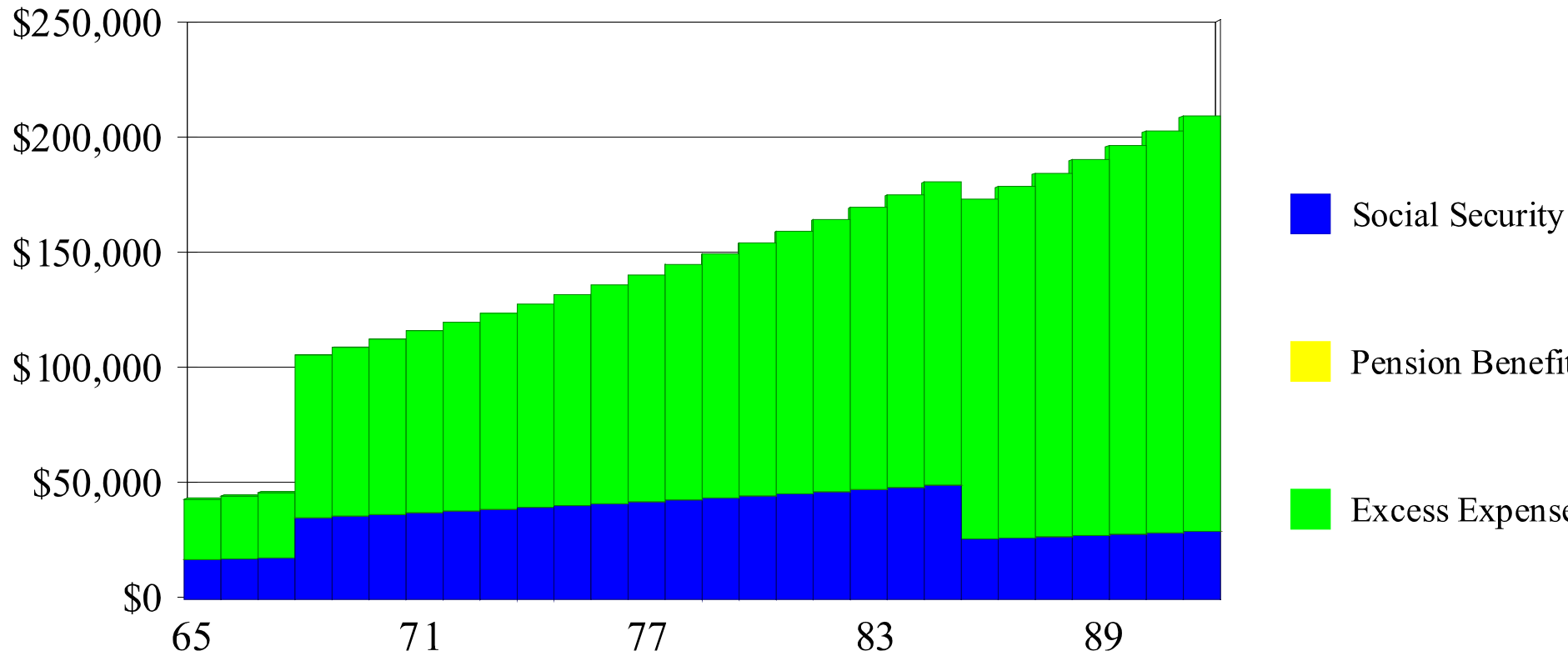
After tax rates of return average 6.12%, with a std. dev. of 8% (95% of values fall between -9.18% and 22.82%).

# Mathematical Model of the Future

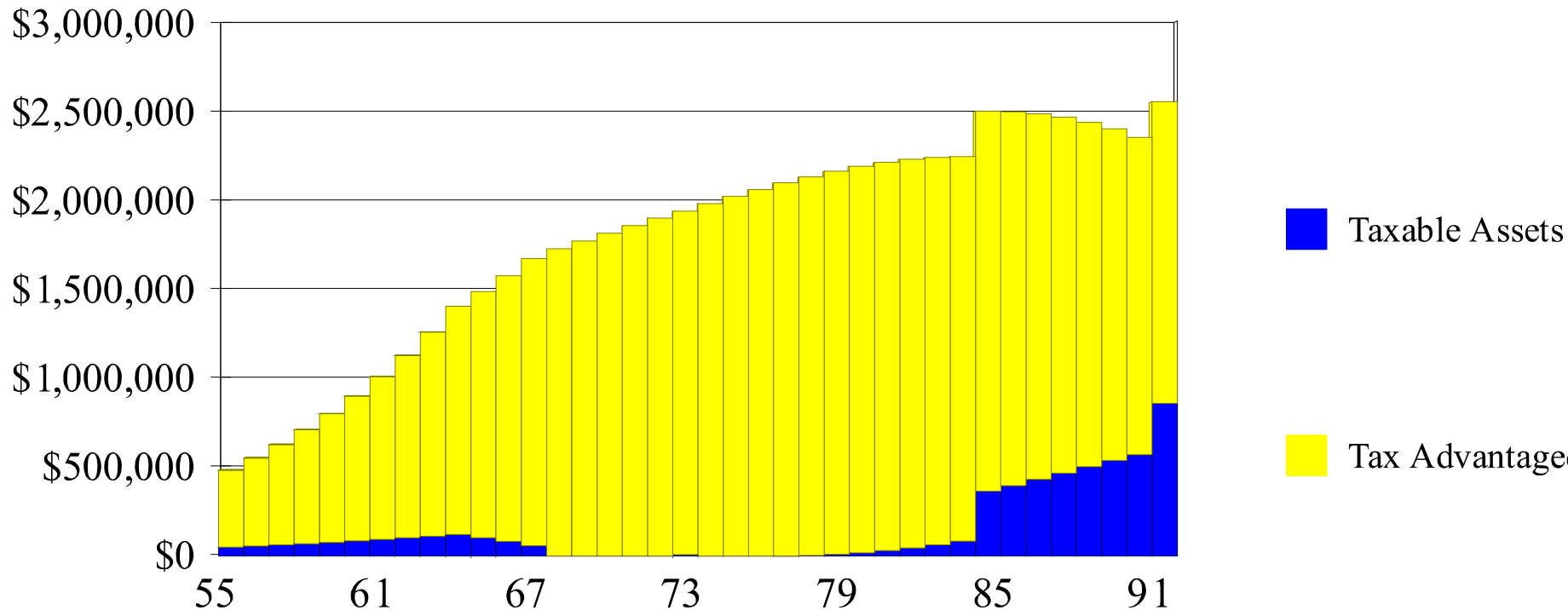
- **Assets, Income, Additions, Growth**
- **Expenses, Withdrawals, Taxes**
- **Pensions, Social Security, Insurance**
- **Plans, Provisions, Special Situations**

These balances and flows may be projected over time and combined into a cohesive model, an approximation of a complex financial life.

# Retirement Expense Projection



# Retirement Asset Projection



# Variables within the Model

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The real world is unpredictable, and things do change:

- **Year by Year Asset Growth**
- **Rate of Return on Deposits**
- **Year by Year Inflation Effects**

# Why Use Monte Carlo?

- **Illustrate variability & uncertainty**
- **Test models in a variable environment**
- **Help design portfolios with less variability**
- **Show the need for ongoing monitoring**
- **Convey a confidence level to the client**
- **Demonstrate unequivocally that the client's financial future is unknown and changeable**

# Change, Fluctuations & Chaos

- **Random Behavior in Natural Systems**
  - tree growth, weather, populations
- **Noise vs. Trends**
  - variations from the norm are normal
- **Chaos is**
  - great complexity, multiple interactive influences, questionable predictability

# Chaos

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- **Patterns indiscernible at one level are often clear at higher levels**
- **Interactions and relationships are complex and subtle**
- **Individual outcomes are unknowable, yet the larger trends and cumulative results may be predictable**

# Simulation Technology

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- **Simulation technology uses simulated chaos to find larger trends and cumulative results of complex systems**
- **Statistical analysis of results can help relate trends to simpler percentage terms**
- **Graphic representations may help illustrate both the technique and the resulting trends and scope of calculations**

# Why Use Simulation Technology?

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- **Illustrates & Communicates Uncertainty**
  - Full Disclosure / Compliance Issues
- **Promote Scheduled Plan Re-evaluation**
  - Annual or Bi-annual Plan Reviews
- **Return/Inflation Sensitivity Measurement**
  - Plan performance evaluation

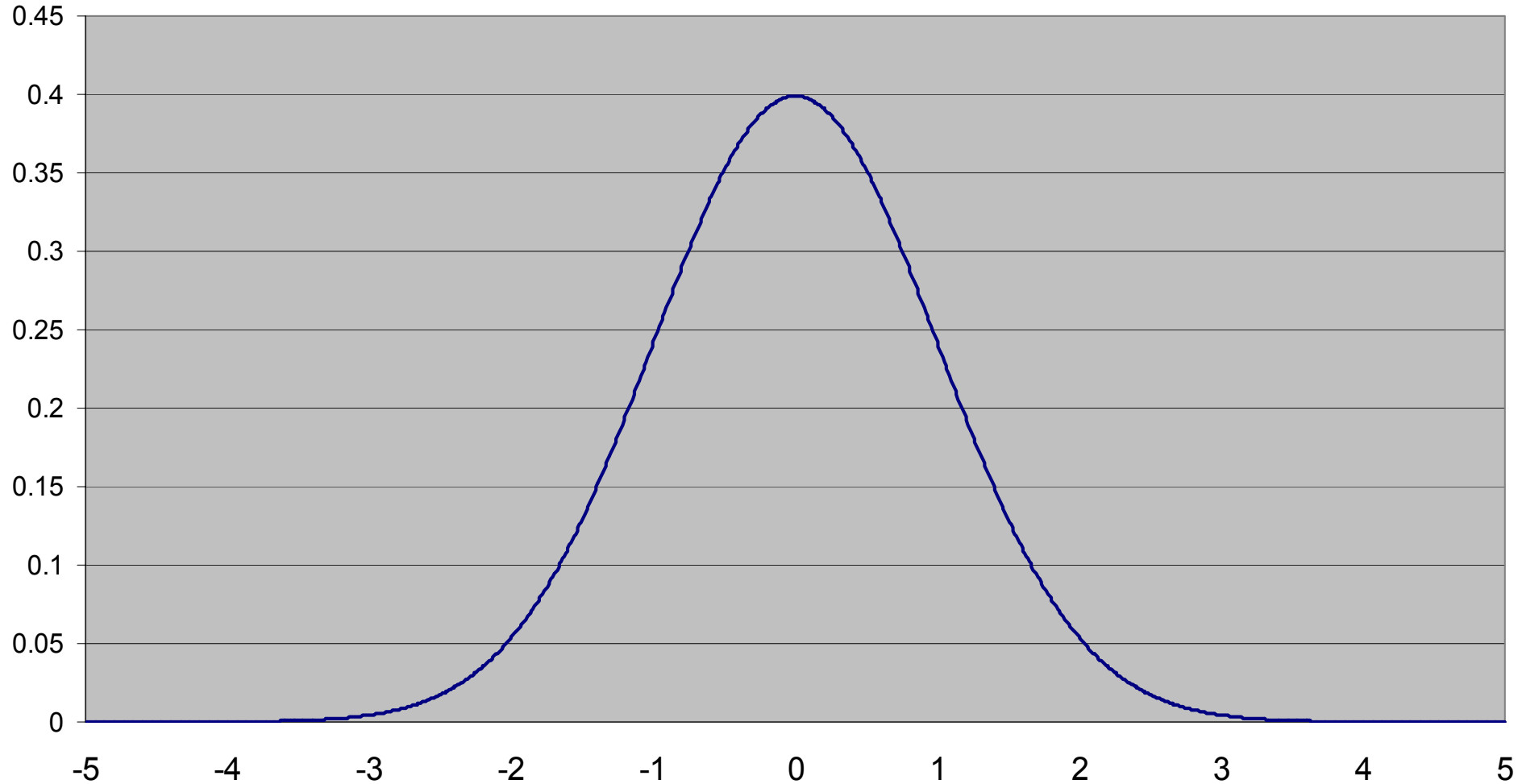
# How to present Simulation

- **State assumptions about the general financial plan, and discuss the results of the average or nominal projections calculated statically**
- **Explain the effects of market and economic environment on the plan's assumptions**
- **Show the Simulation results as a representation of a potential range of actual results based on changing and unpredictable markets**
- **Discuss comfort level and probable outcomes**

# Standard Normal Density Function

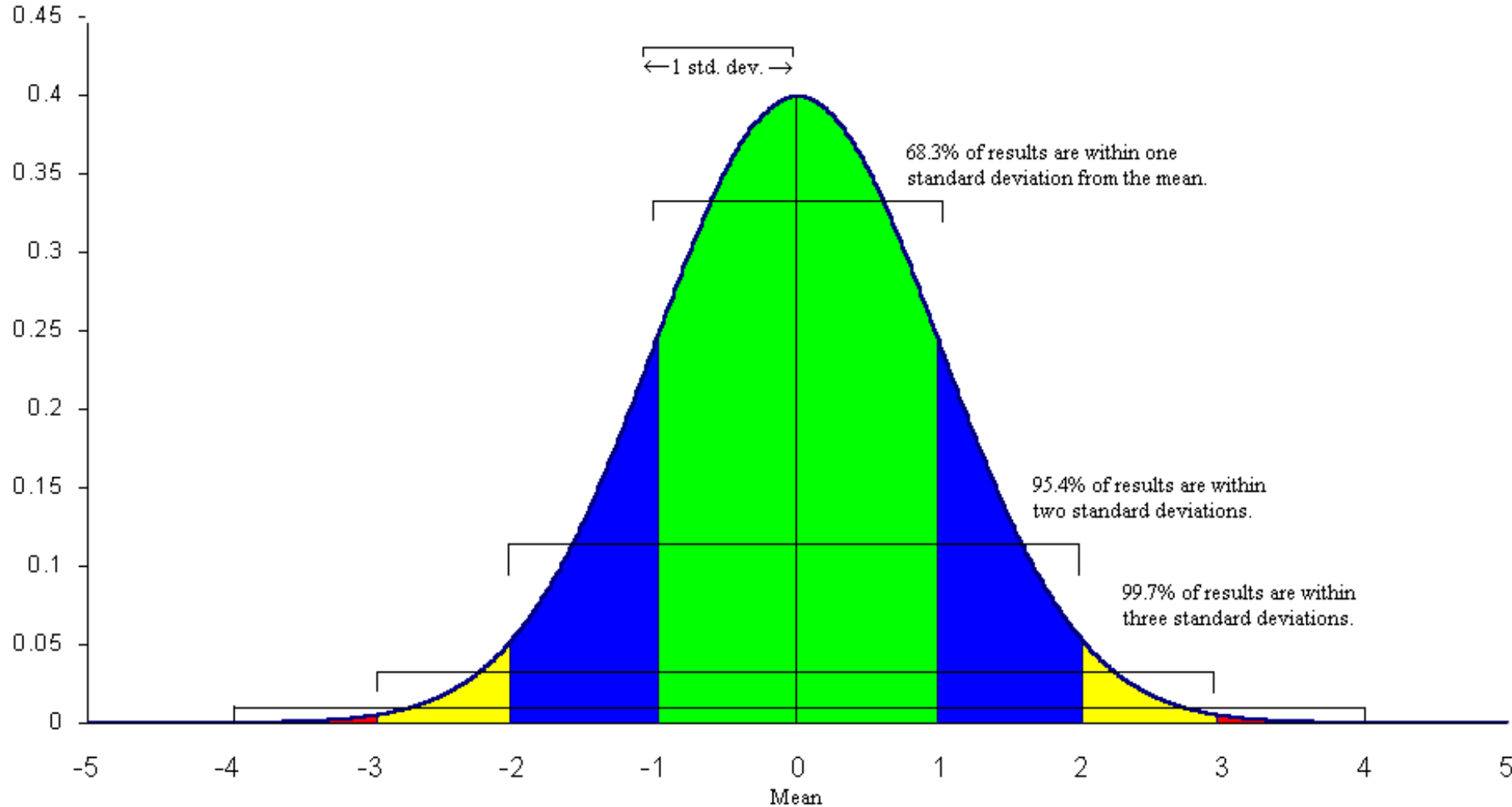
Graph of the Standard Normal Density Function

$$g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

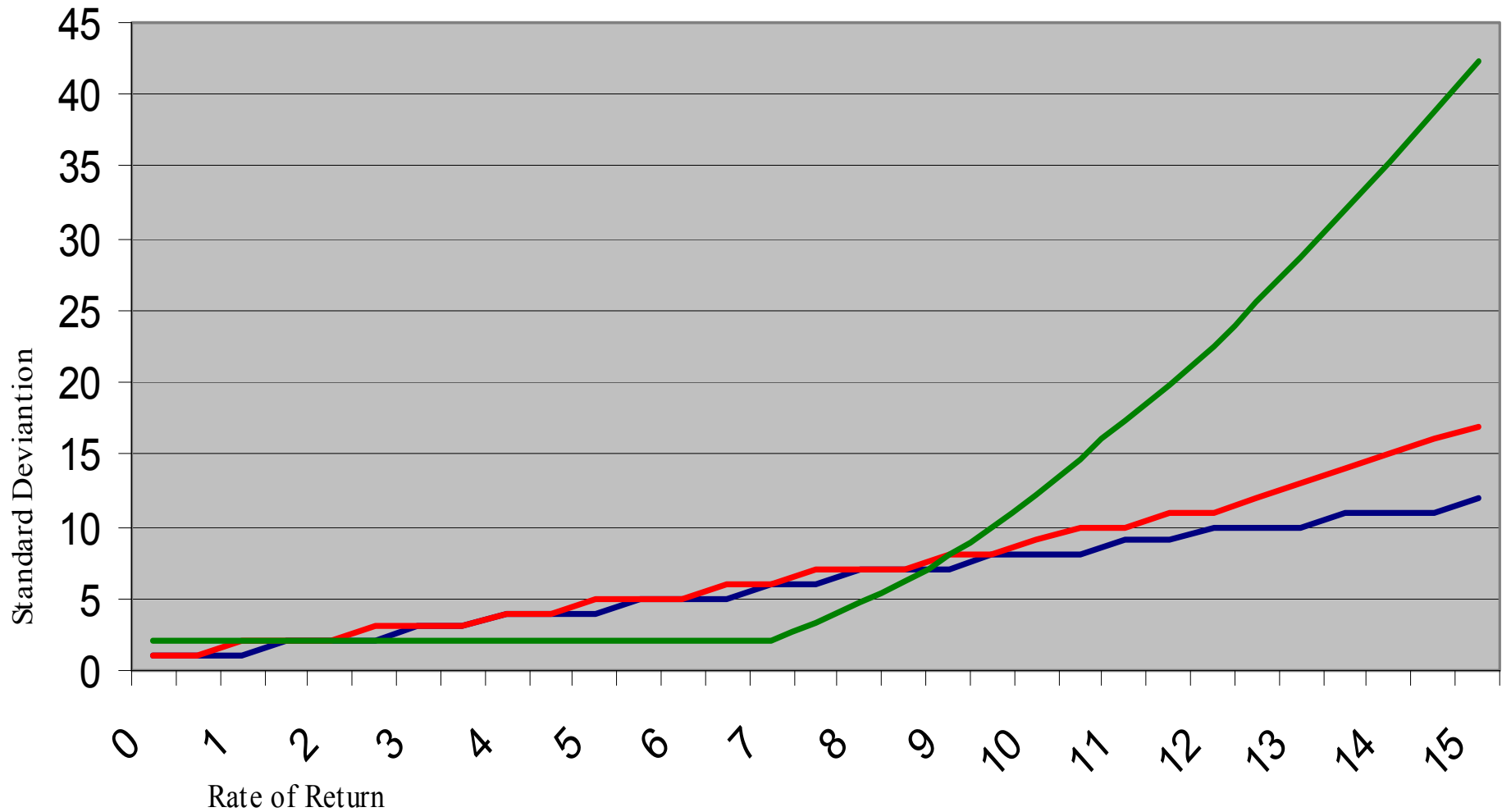


# Normal Distribution & Standard Deviation

Standard Deviation is a way to describe variance in a set of numbers. Monte Carlo Simulation uses normally distributed random numbers to create model trials. The standard deviation chosen for the simulation determines the range of possible and most probable results for each period. This illustration of normally distributed numbers shows how results are dispersed.



# Standard Deviation Functions



— .75\*ROR

— Mtree New

— Strong

# Portfolio Standard Deviation Calculations

A technique to calculate the standard deviation of a mixed portfolio of 30% Bonds, 40% MidCap fund and 30% SmallCap fund.

Rates of Return: Bonds = 4.5%, MidCap = 13.0%, SmallCap = 18.6%

Std Dev : Bonds = 3.0%, MidCap = 11.0%, SmallCap = 20.0%

Portfolio = 30% Bonds + 40% MidCap + 30% SmallCap

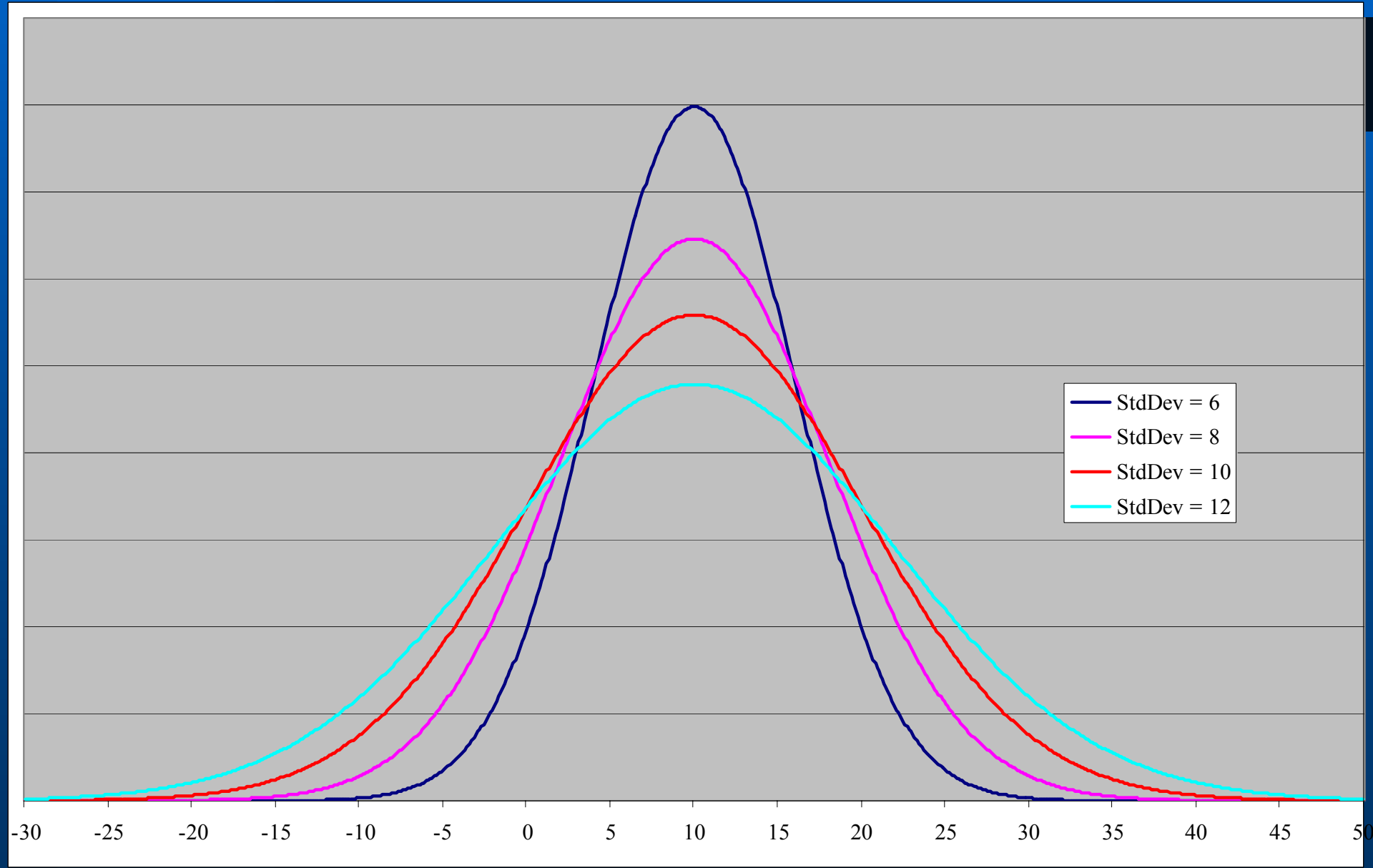
Rates of Return(Portfolio) =  $(.30)(4.5\%) + (.40)(13\%) + (.30)(18.6\%) = 12.13\%$

Variance(Portfolio) =  $(.30)^2 * (3)^2 + (.40)^2 * (11)^2 + (.30)^2 * (20)^2$

Variance(Portfolio) =  $(.09) * (9) + (.16) * (121) + (.09) * (400) = 56.17$

Std Dev (Portfolio) = Square Root (56.17) = 7.49%

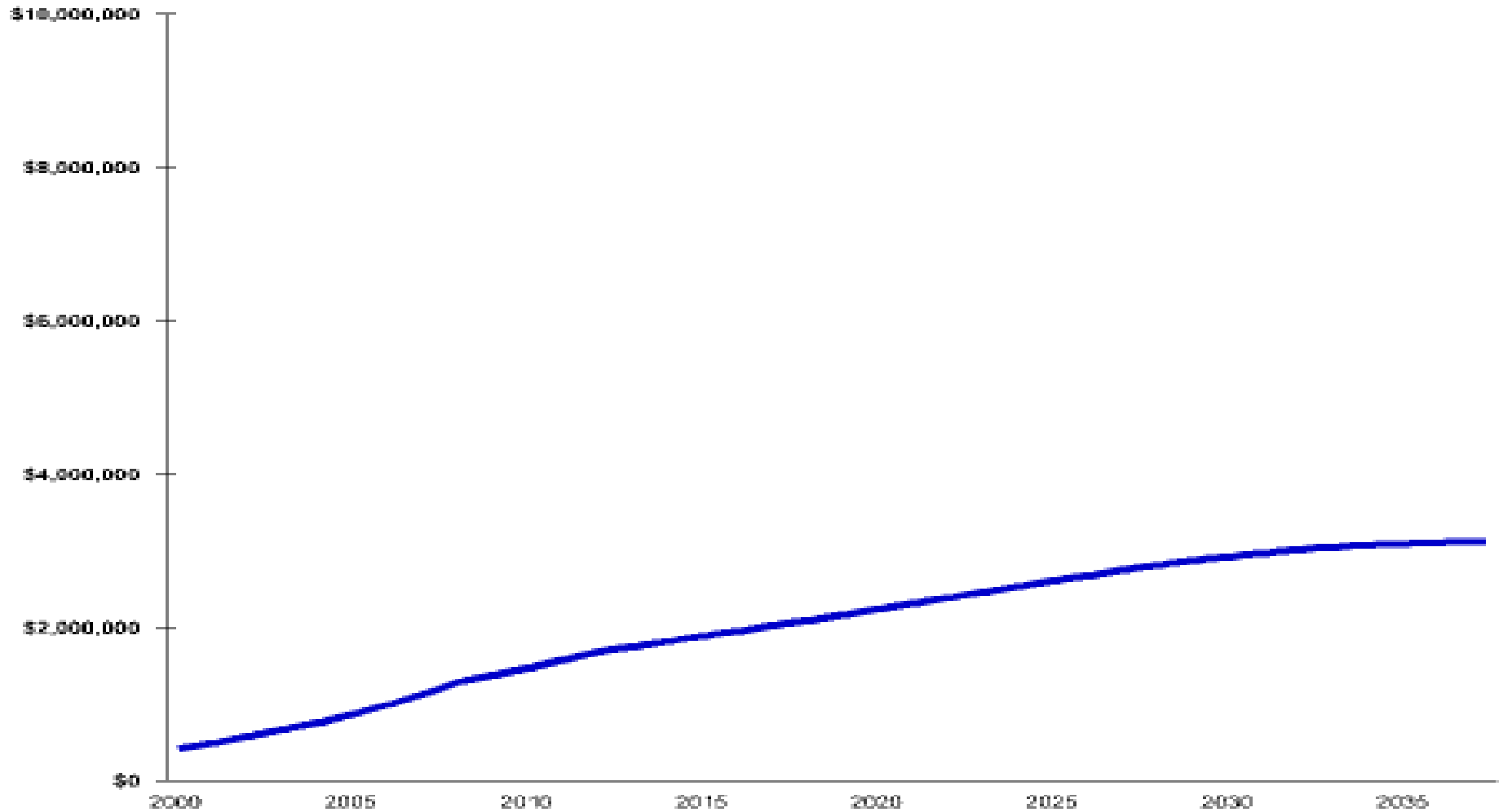
# Effects of Standard Deviations



# Standard Deviation: Zero

Results of 1000 Simulations:	Percentage of projections above zero	100%
	Retirement Projection Estimate	\$3,123,022
	Minimum Monte Carlo projection	\$3,123,022
	Average Monte Carlo projection	\$3,123,022
	Maximum Monte Carlo projection	\$3,123,022

## Monte Carlo Simulation Results



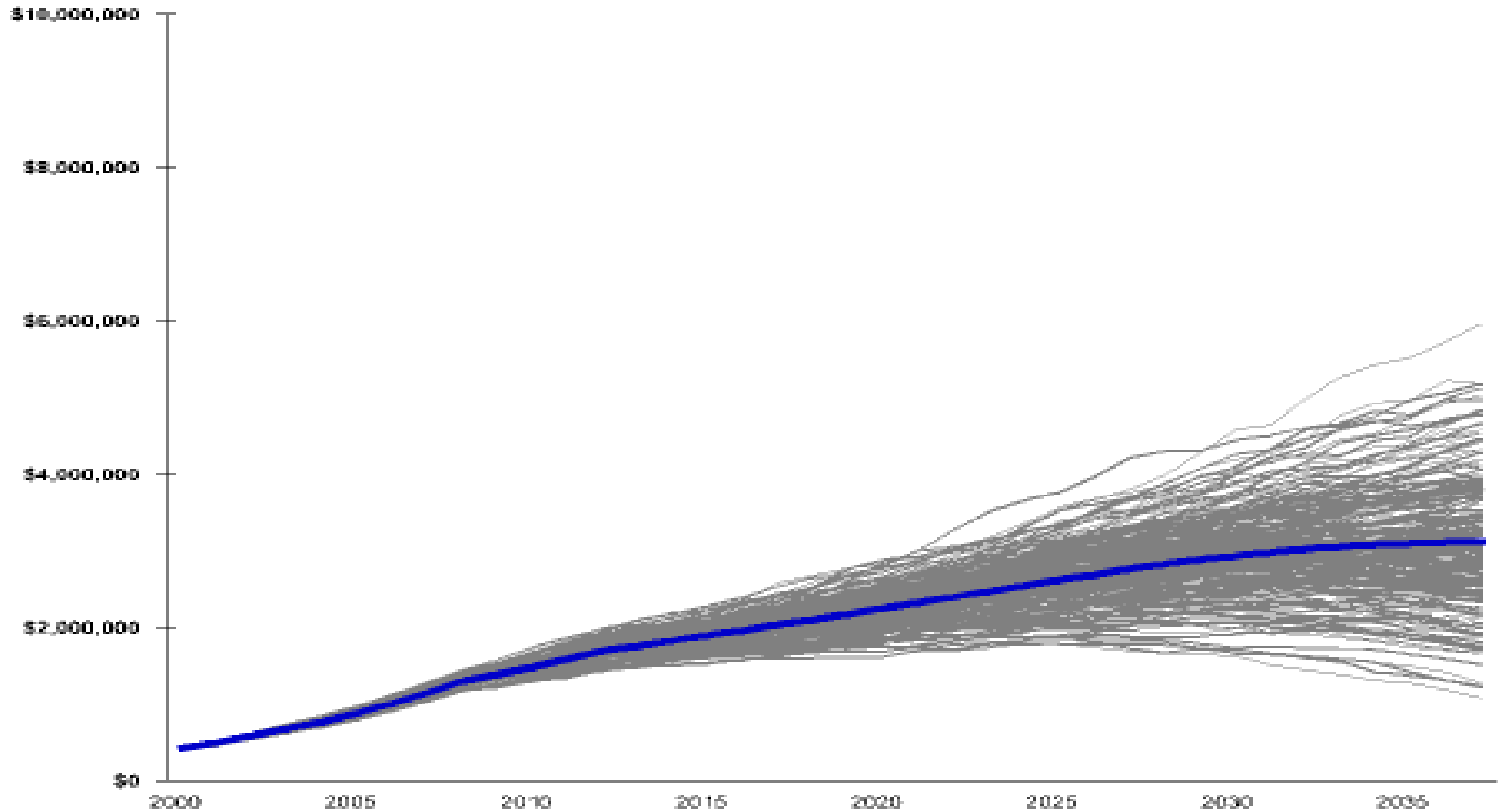
\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.82%, with a std. dev. of 0% (95% of values fall between 6.82% and 6.82%).

# Standard Deviation: Two

Results of 1000 Simulations:	Percentage of projections above zero	100%
	Retirement Projection Estimate	\$3,123,022
	Minimum Monte Carlo projection	\$653,639
	Average Monte Carlo projection	\$3,129,001
	Maximum Monte Carlo projection	\$6,303,086

## Monte Carlo Simulation Results



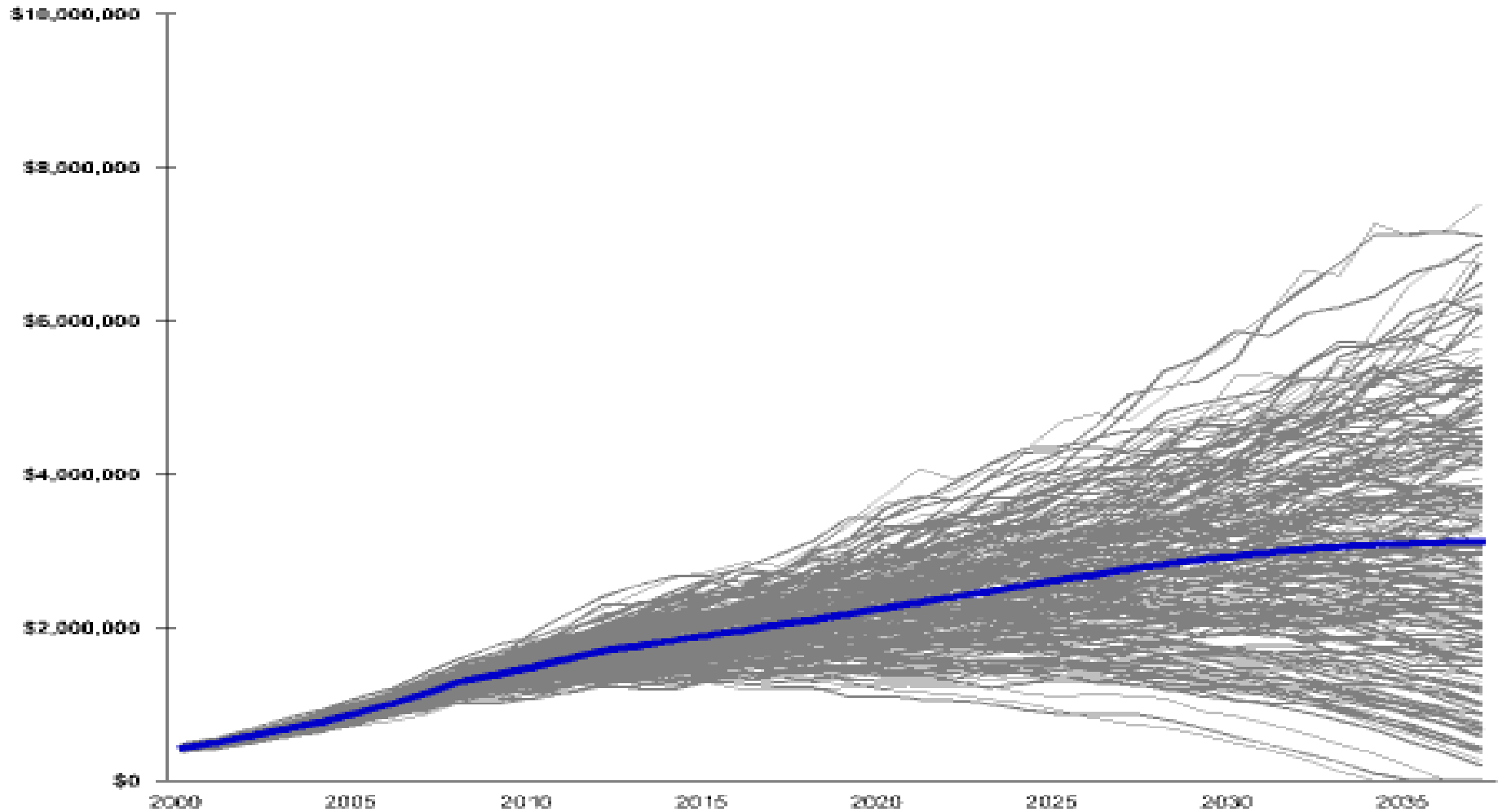
\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.82%, with a std. dev. of 2% (95% of values fall between 2.82% and 10.82%).

# Standard Deviation: Four

Results of 1000 Simulations:	Percentage of projections above zero	99%
	Retirement Projection Estimate	\$3,123,022
	Minimum Monte Carlo projection	\$0
	Average Monte Carlo projection	\$3,151,228
	Maximum Monte Carlo projection	\$10,119,274

## Monte Carlo Simulation Results



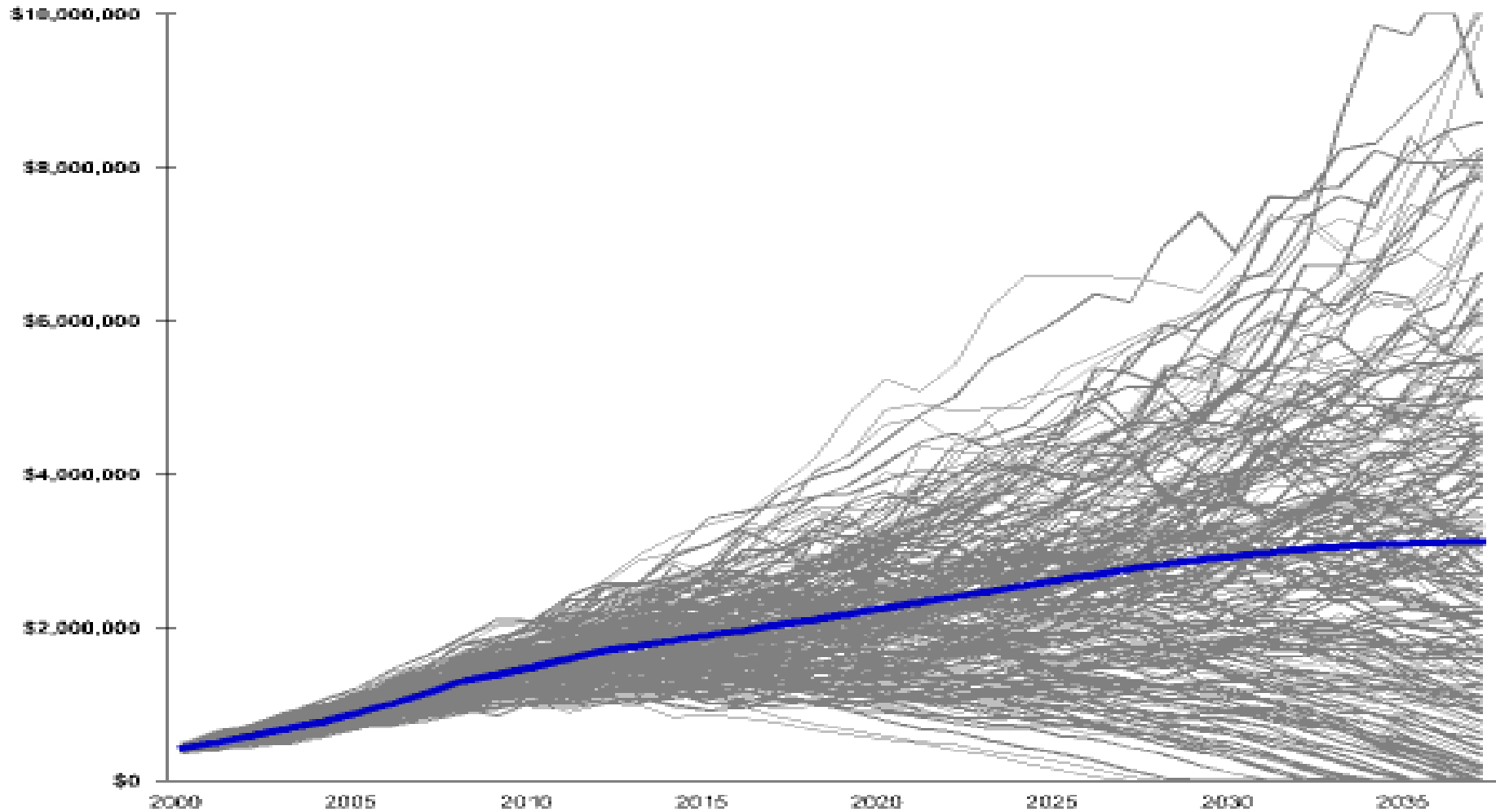
\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.82%, with a std. dev. of 4% (95% of values fall between -1.18% and 14.82%).

# Standard Deviation: Seven

Results of 1000 Simulations:	Percentage of projections above zero	86%
	Retirement Projection Estimate	\$3,123,022
	Minimum Monte Carlo projection	\$0
	Average Monte Carlo projection	\$3,030,590
	Maximum Monte Carlo projection	\$19,653,719

## Monte Carlo Simulation Results



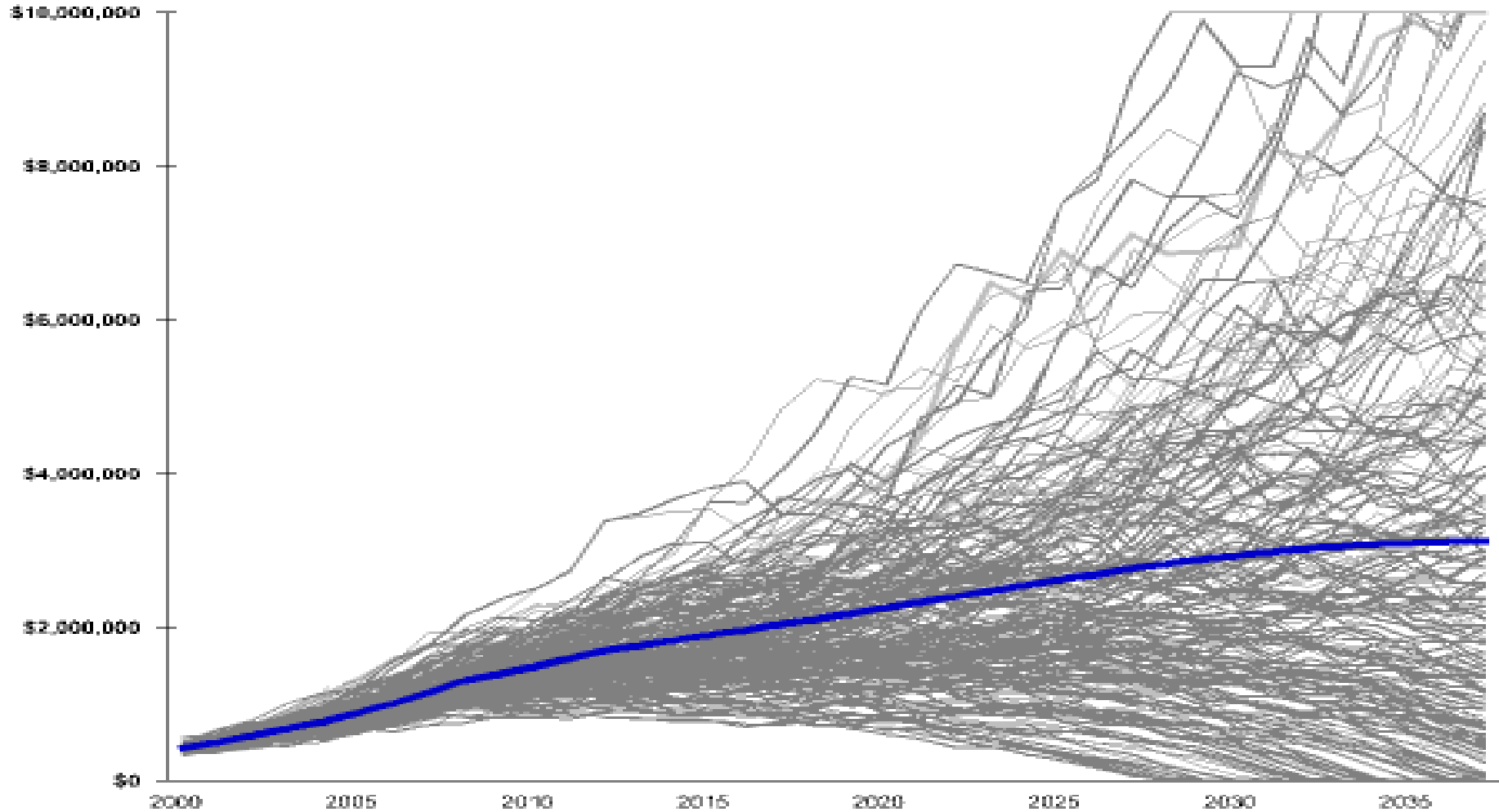
\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.82%, with a std. dev. of 7% (95% of values fall between -7.18% and 20.82%).

# Standard Deviation: Eight

Results of 1000 Simulations:	Percentage of projections above zero	84%
	Retirement Projection Estimate	\$3,123,022
	Minimum Monte Carlo projection	\$0
	Average Monte Carlo projection	\$3,165,938
	Maximum Monte Carlo projection	\$20,351,776

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

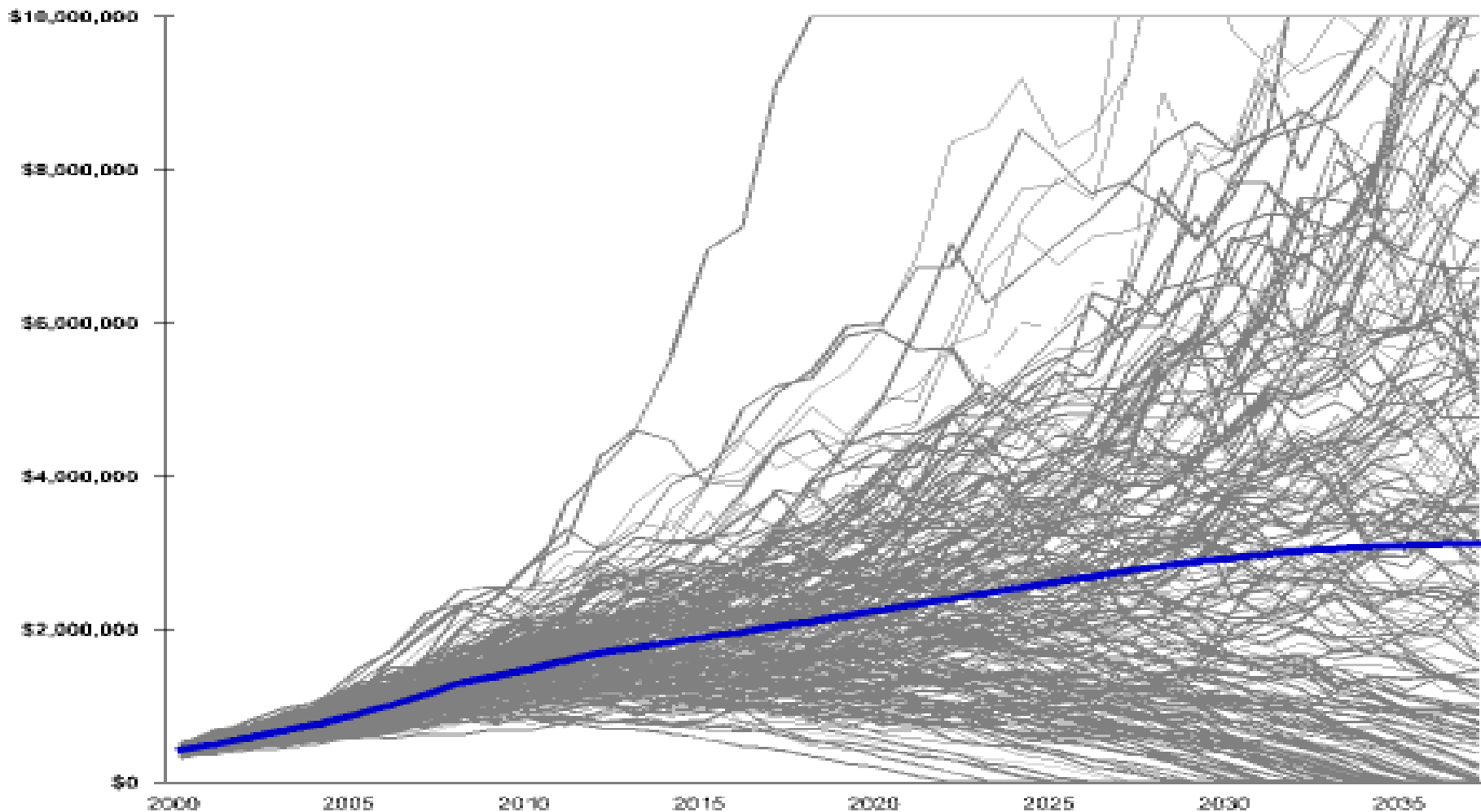
After tax rates of return average 6.82%, with a std. dev. of 8% (95% of values fall between -9.18% and 22.82%).

# Standard Deviation: Ten

Results of 1000 Simulations:

Percentage of projections above zero	75%
Retirement Projection Estimate	\$3,123,022
Minimum Monte Carlo projection	\$0
Average Monte Carlo projection	\$3,368,356
Maximum Monte Carlo projection	\$35,466,671

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.82%, with a std. dev. of 10% (95% of values fall between -13.18% and 26.82%).

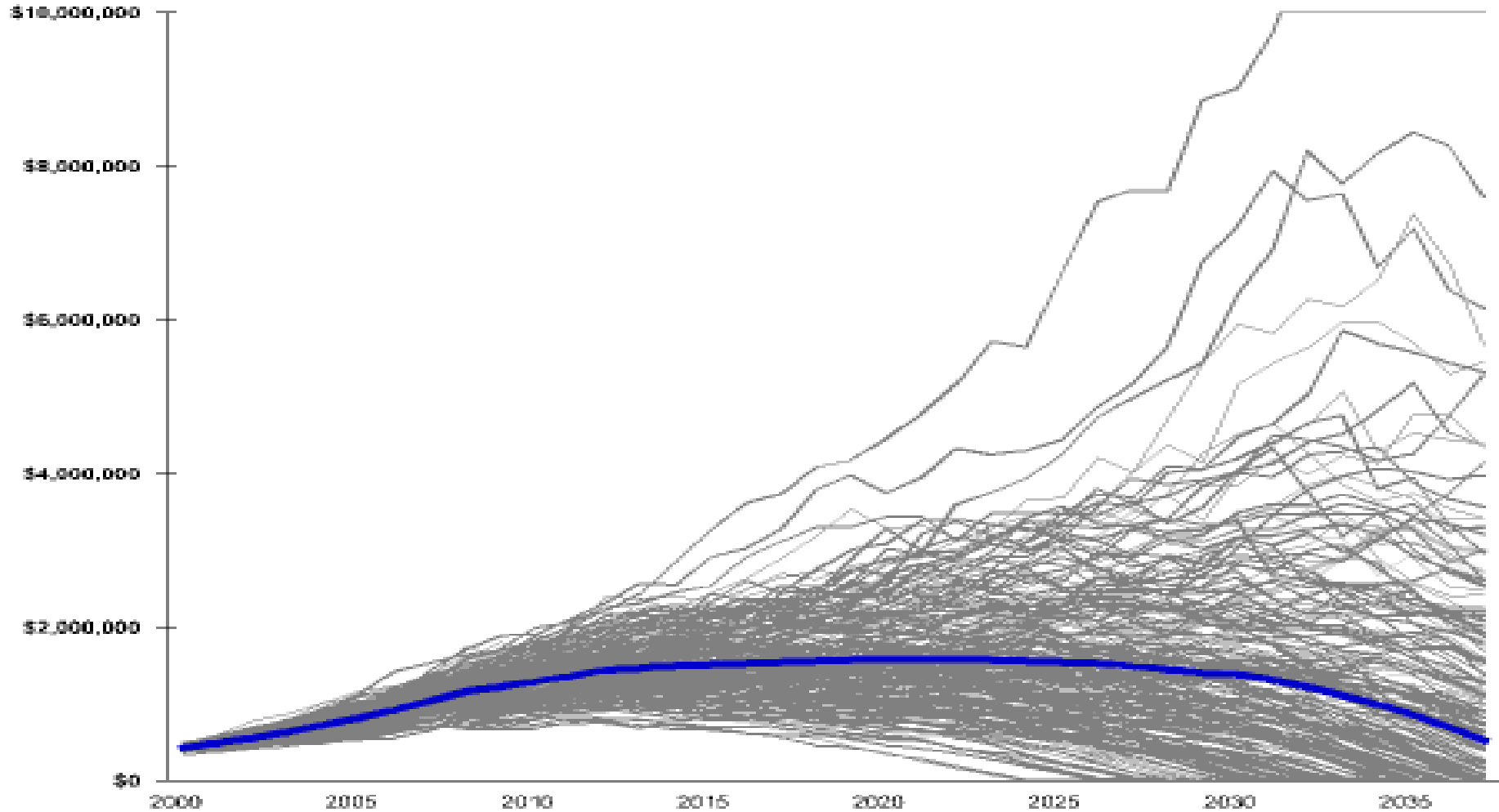
## Plan Analysis: Evaluating projections during uncertain conditions

- **Measure plan results, and evaluate the probability of plan success through life expectancy**
- **Modify the plan to adjust for uncertainty and provide a comfortable level of plan performance**
- **Consider effects of portfolio allocation on risk and uncertainty**
- **Review plan performance over time**

# Analysis: Starting projection

Results of 1000 Simulations:	Percentage of projections above zero	59%
	Retirement Projection Estimate	\$526,036
	Minimum Monte Carlo projection	\$0
	Average Monte Carlo projection	\$932,570
	Maximum Monte Carlo projection	\$14,143,859

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.12%, with a std. dev. of 7% (95% of values fall between -7.88% and 20.12%).

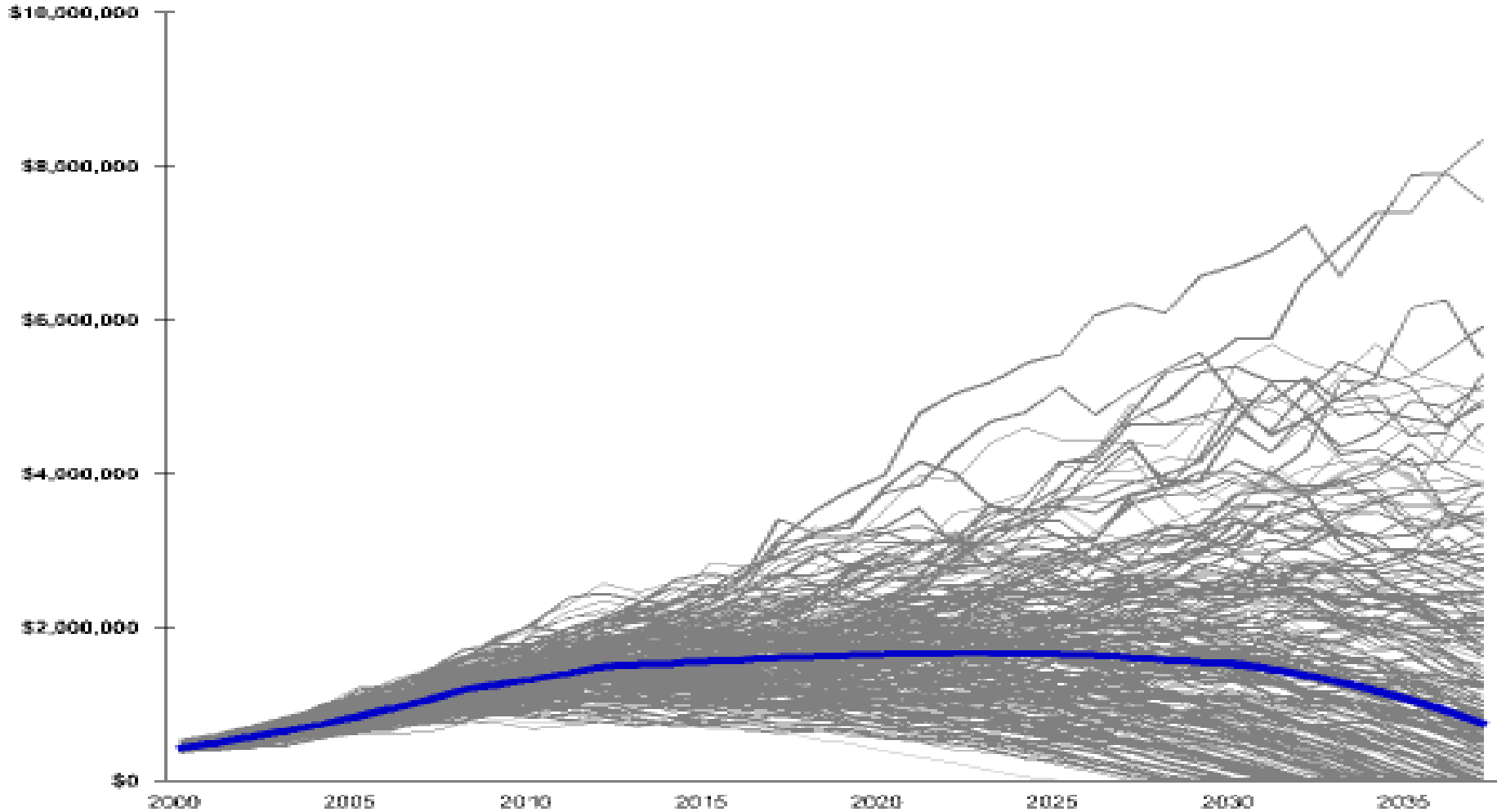
# Analysis: Additional Savings

+ \$3000

Results of 1000 Simulations:

Percentage of projections above zero	60%
Retirement Projection Estimate	\$751,836
Minimum Monte Carlo projection	\$0
Average Monte Carlo projection	\$1,112,433
Maximum Monte Carlo projection	\$10,275,389

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.12%, with a std. dev. of 7% (95% of values fall between -7.88% and 20.12%).

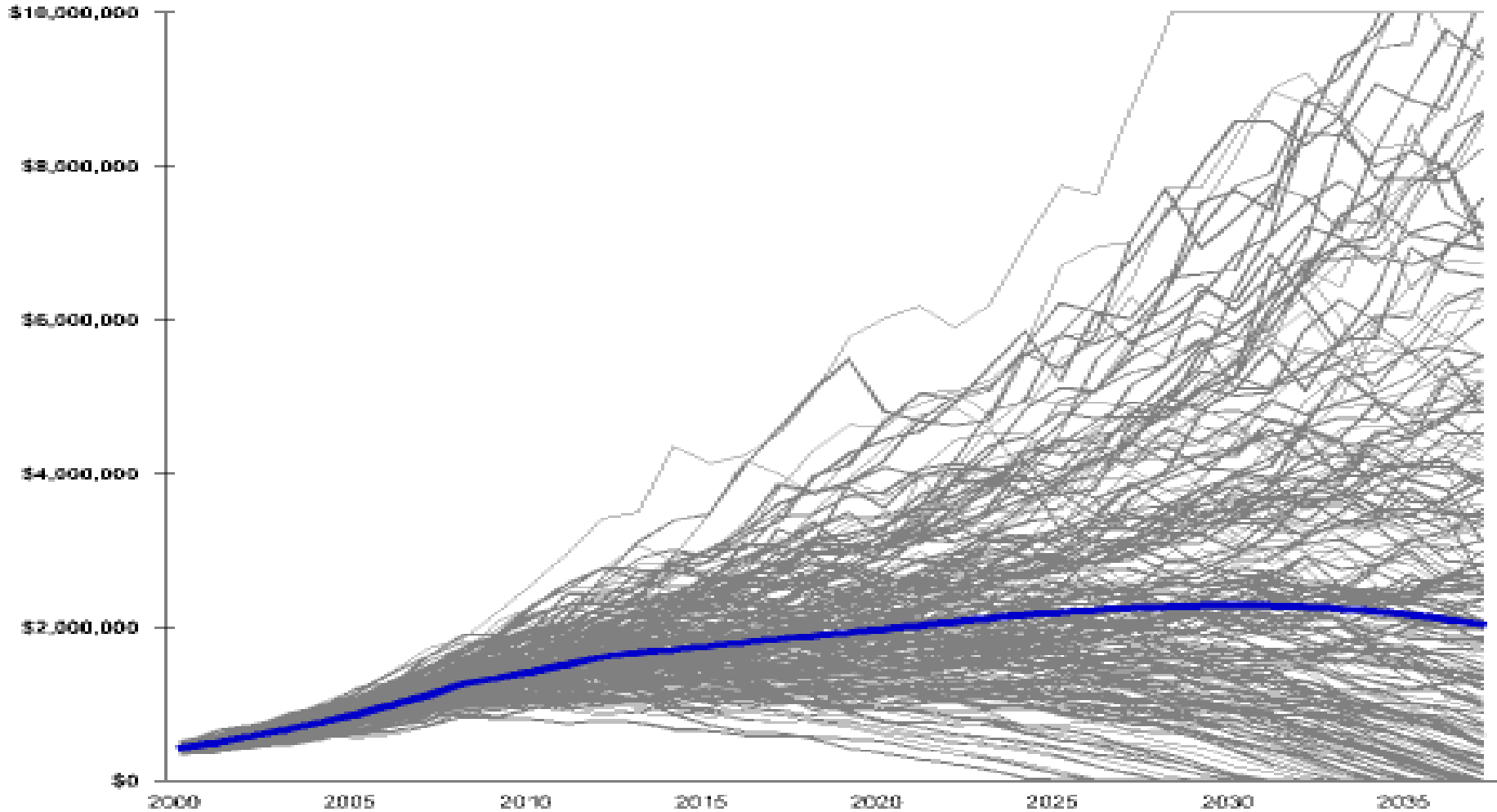
# Analysis: Additional Savings

+ \$6000

Results of 1000 Simulations:

Percentage of projections above zero	80%
Retirement Projection Estimate	\$2,037,342
Minimum Monte Carlo projection	\$0
Average Monte Carlo projection	\$2,403,443
Maximum Monte Carlo projection	\$22,440,606

## Monte Carlo Simulation Results



\* The bold line is the estimated retirement projection using fixed rates.

After tax rates of return average 6.12%, with a std. dev. of 7% (95% of values fall between -7.88% and 20.12%).